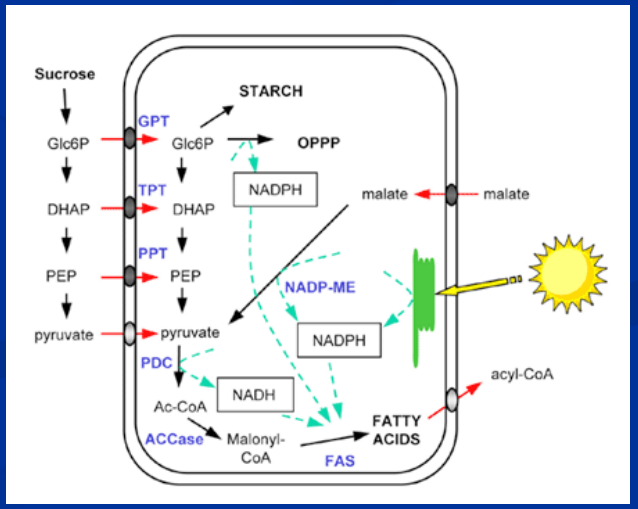
INSTITUTO DE FÍSICA
Universidade Federal Fluminense

Redes complexas

Marcio Argollo de Menezes
Universidade Federal Fluminense
Niterói, Rio de Janeiro



Redes: um paradigma de sistemas interagentes

Redes

- Físicas: Estruturas estáticas/dinâmicas
- Relacionais: Sociais, econômicas, tecnológicas

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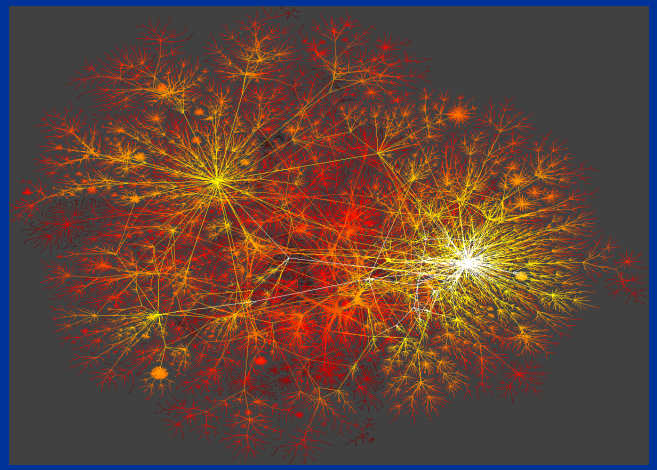
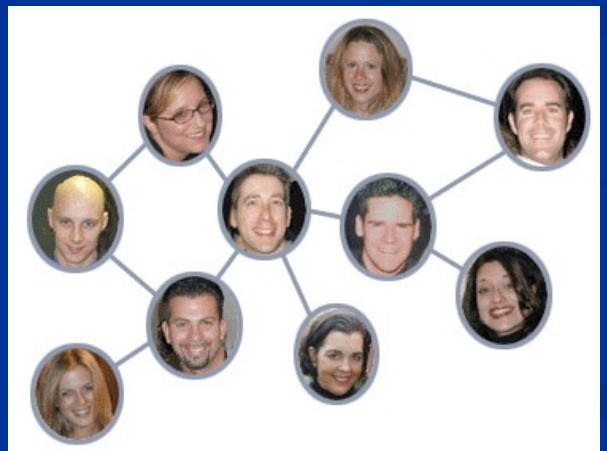
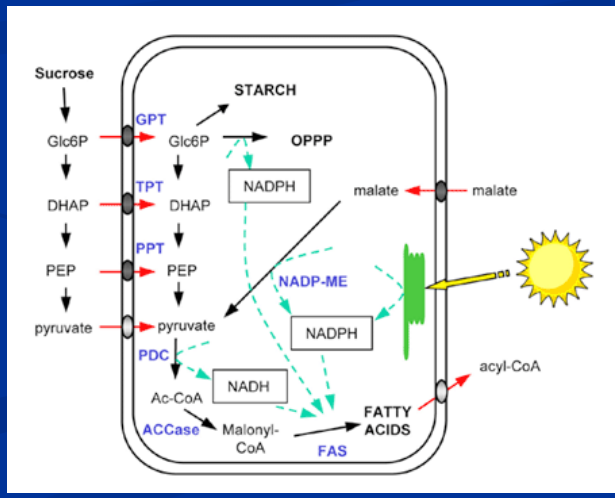
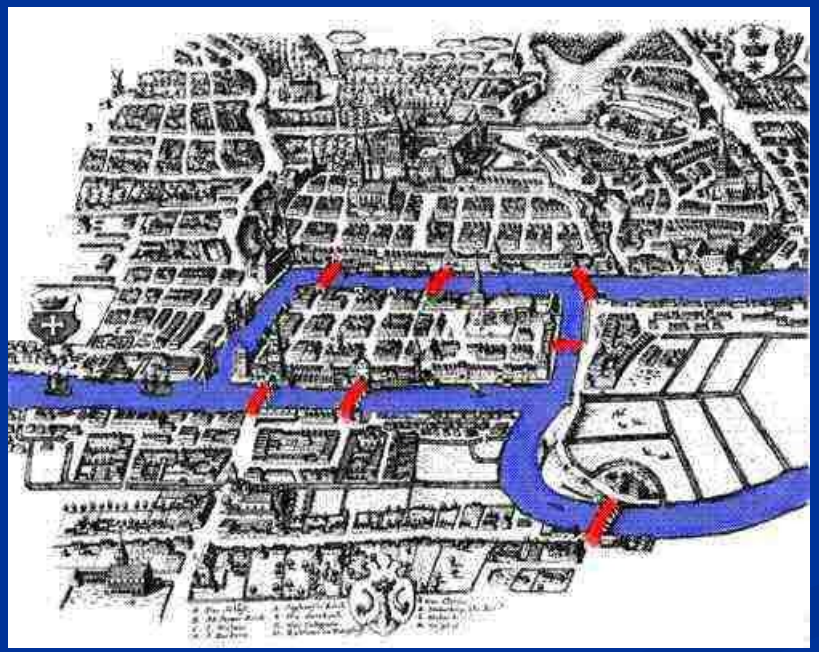






Redes: um paradigma de sistemas interagentes

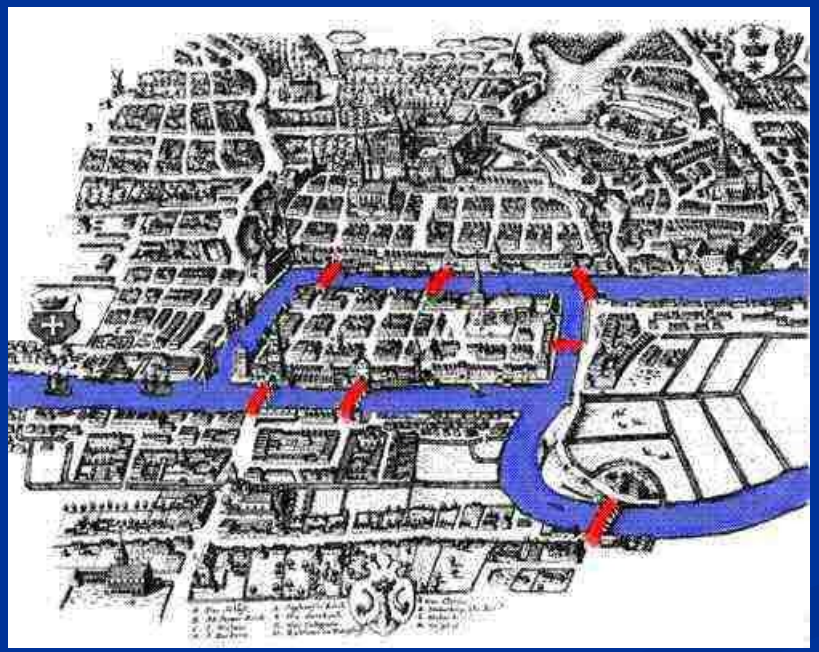
Grafos: Euler (1736) e as “Sete Pontes de Koenigsberg”

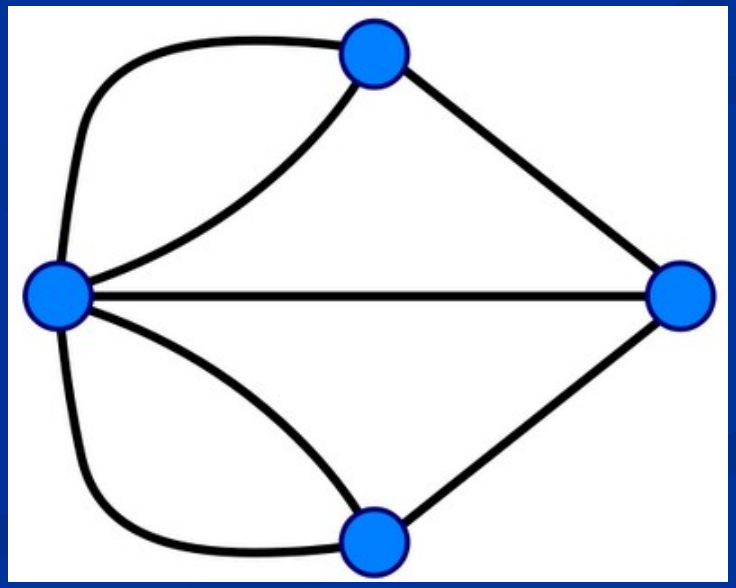


Redes: um paradigma de sistemas interagentes

Grafos: Euler (1736) e as “Sete Pontes de Koenigsberg”

- Abstração do problema
- Discriminação de elementos relevantes
- Redução de complexidade sem perda de informação relevante

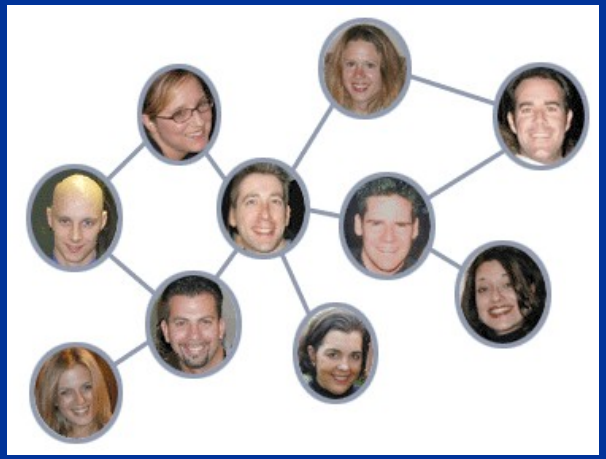




A experiência de Milgram

Stanley Milgram "The Small World Problem", Psychology Today 2: 60-67 (1967).
http://en.wikipedia.org/wiki/Small_world_phenomenon

- Remetentes aleatoriamente escolhidos na costa leste dos EUA
- Destinatário na costa oeste
- Se remetente conhece o destinatário pessoalmente, entrega a carta
- Se não conhecer, entrega para amigo que possa conhecer o destinatário



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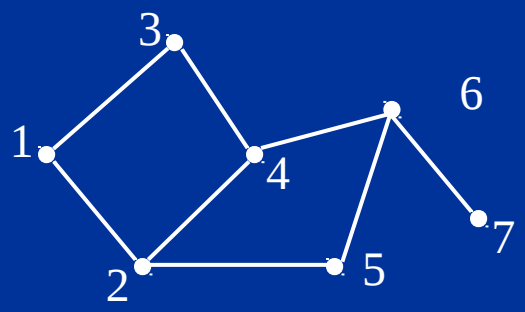

$l_{15}=2$ [1→2→5]

$l_{17}=4$ [1→3→4→6 → 7]

… $< l >$ = ??

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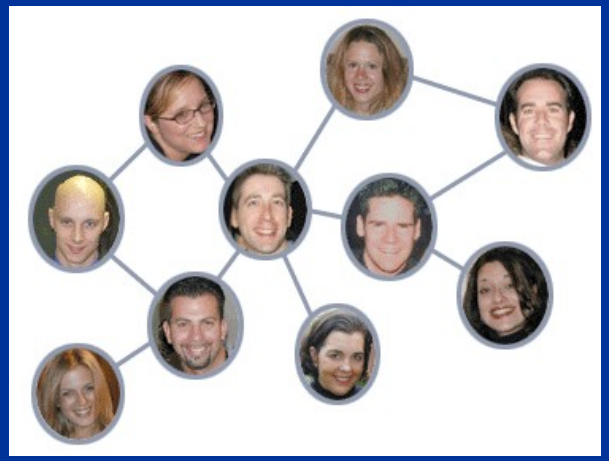

"Six degrees of freedom"

$N \sim 10^6$

$\langle l \rangle \sim 6$

	network	type	n	m	z	ℓ
social	film actors	undirected	449 913	25 516 482	113.43	3.48
	company directors	undirected	7 673	55 392	14.44	4.60
	math coauthorship	undirected	253 339	496 489	3.92	7.57
	physics coauthorship	undirected	52 909	245 300	9.27	6.19
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92
	telephone call graph	undirected	47 000 000	80 000 000	3.16	
	email messages	directed	59 912	86 300	1.44	4.95
	email address books	directed	16 881	57 029	3.38	5.22
	student relationships	undirected	573	477	1.66	16.01
	sexual contacts	undirected	2 810			
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18
	citation network	directed	783 339	6 716 198	8.57	
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87
	word co-occurrence	undirected	460 902	17 000 000	70.13	
technological	Internet	undirected	10 697	31 992	5.98	3.31
	power grid	undirected	4 941	6 594	2.67	18.99
	train routes	undirected	587	19 603	66.79	2.16
	software packages	directed	1 439	1 723	1.20	2.42
	software classes	directed	1 377	2 213	1.61	1.51
	electronic circuits	undirected	24 097	53 248	4.34	11.05
	peer-to-peer network	undirected	880	1 296	1.47	4.28
biological	metabolic network	undirected	765	3 686	9.64	2.56
	protein interactions	undirected	2 115	2 240	2.12	6.80
	marine food web	directed	135	598	4.43	2.05
	freshwater food web	directed	92	997	10.84	1.90
	neural network	directed	307	2 359	7.68	3.97

Grafos Aleatórios de Erdös-Rényi

P. Erdos and A. Renyi. On random graphs. Publ. Math. Debrecen, 6:290--297, 1959.

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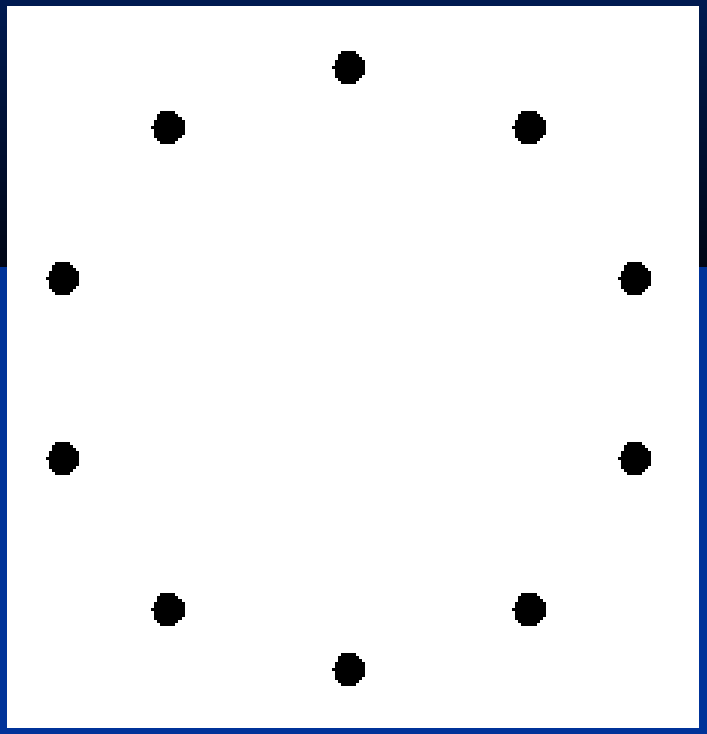

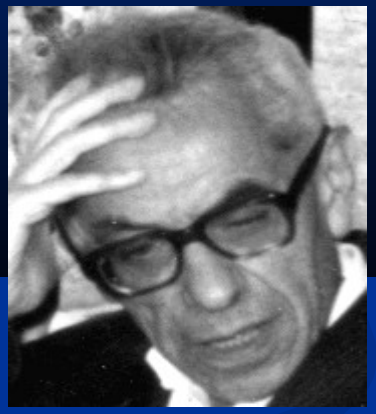

Pál Erdös
(1913-1996)

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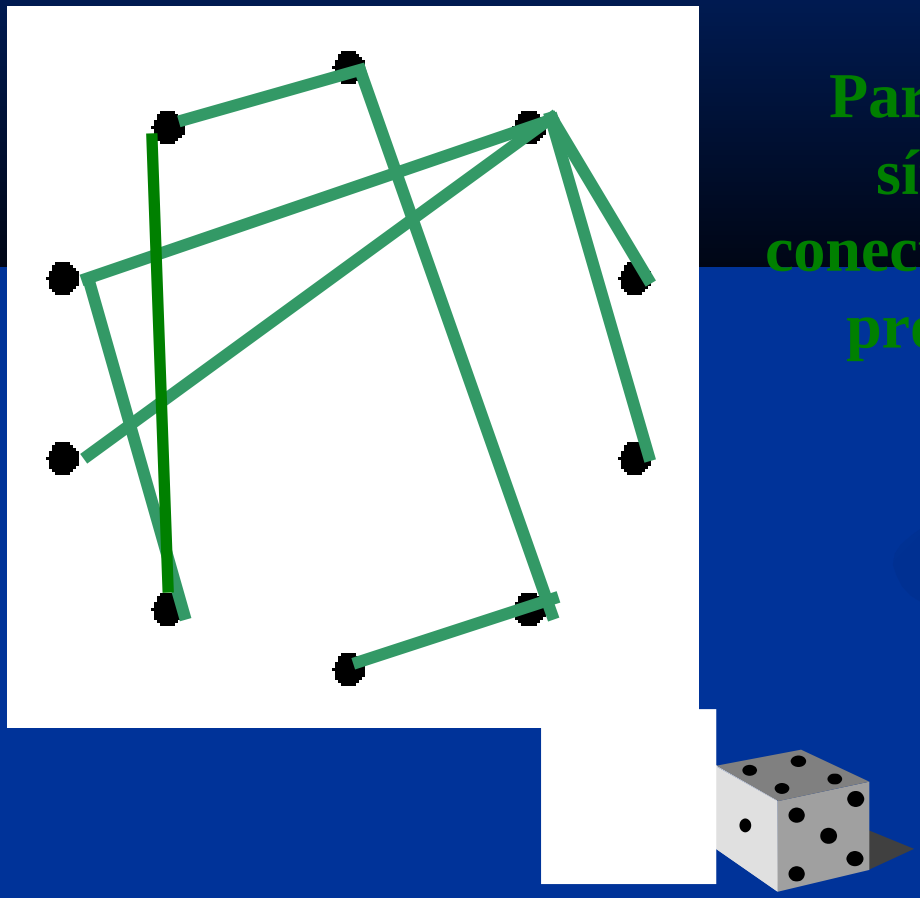
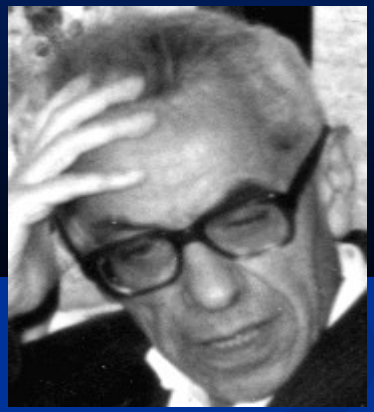


Pares de sítios conectados c/ prob. p

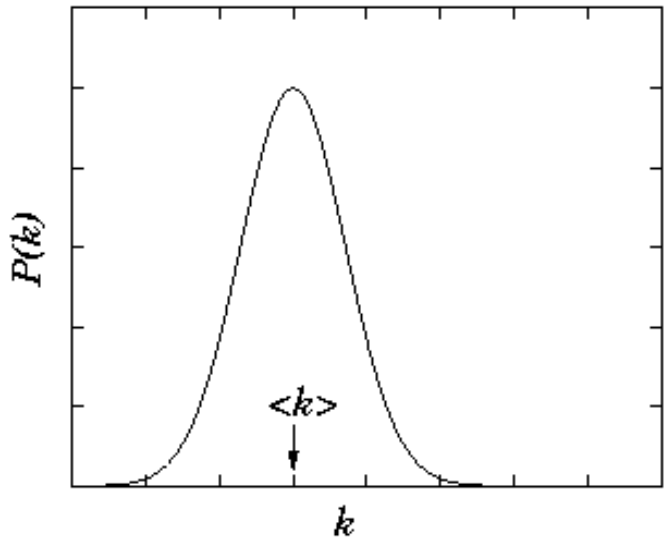


Pál Erdös
(1913-1996)

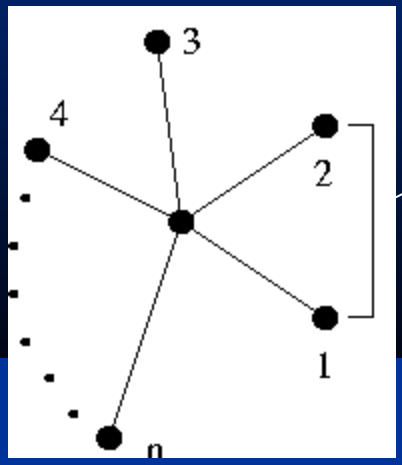
- The $G_{n,p}$ model
  - n : the number of vertices
  - $0 \leq p \leq 1$
  - for each pair (i,j), generate the edge (i,j) independently with probability p

Poisson distribution

$$p(k) = \frac{z^k}{k!} e^{-z}, \quad z = \langle k \rangle = Np$$



E o amigo do amigo?



Probability to be connected = C

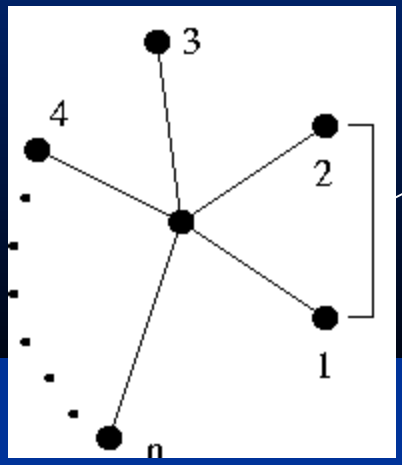
$$C = \frac{\text{\# of links between 1,2,…n neighbors}}{n(n-1)/2}$$

Na rede aleatória

$C \sim p = \langle k \rangle / N$

Loops são improváveis! Localmente temos árvores…

E o amigo do amigo?



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Na rede aleatória
$C \sim p = \langle k \rangle / N$

Em redes empíricas
$C >> 1/N$

Table 1: Clustering coefficients, C, for a number of different networks; n is the number of node, z is the mean degree. Taken from [146].

Network	n	z	C measured	C for random graph
Internet [153]	6,374	3.8	0.24	0.00060
World Wide Web (sites) [2]	153,127	35.2	0.11	0.00023
power grid [192]	4,941	2.7	0.080	0.00054
biology collaborations [140]	1,520,251	15.5	0.081	0.000010
mathematics collaborations [141]	253,339	3.9	0.15	0.000015
film actor collaborations [149]	449,913	113.4	0.20	0.00025
company directors [149]	7,673	14.4	0.59	0.0019
word co-occurrence [90]	460,902	70.1	0.44	0.00015
neural network [192]	282	14.0	0.28	0.049
metabolic network [69]	315	28.3	0.59	0.090
food web [138]	134	8.7	0.22	0.065

- The $G_{n,p}$ model
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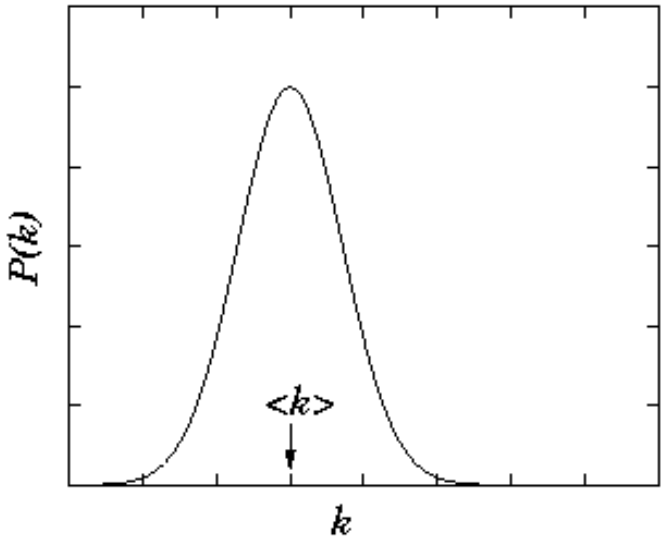
$$p(k) = \frac{z^k}{k!} e^{-z}, \quad z = \langle k \rangle = Np$$

$$C \sim \frac{\langle k \rangle}{N} \rightarrow 0$$

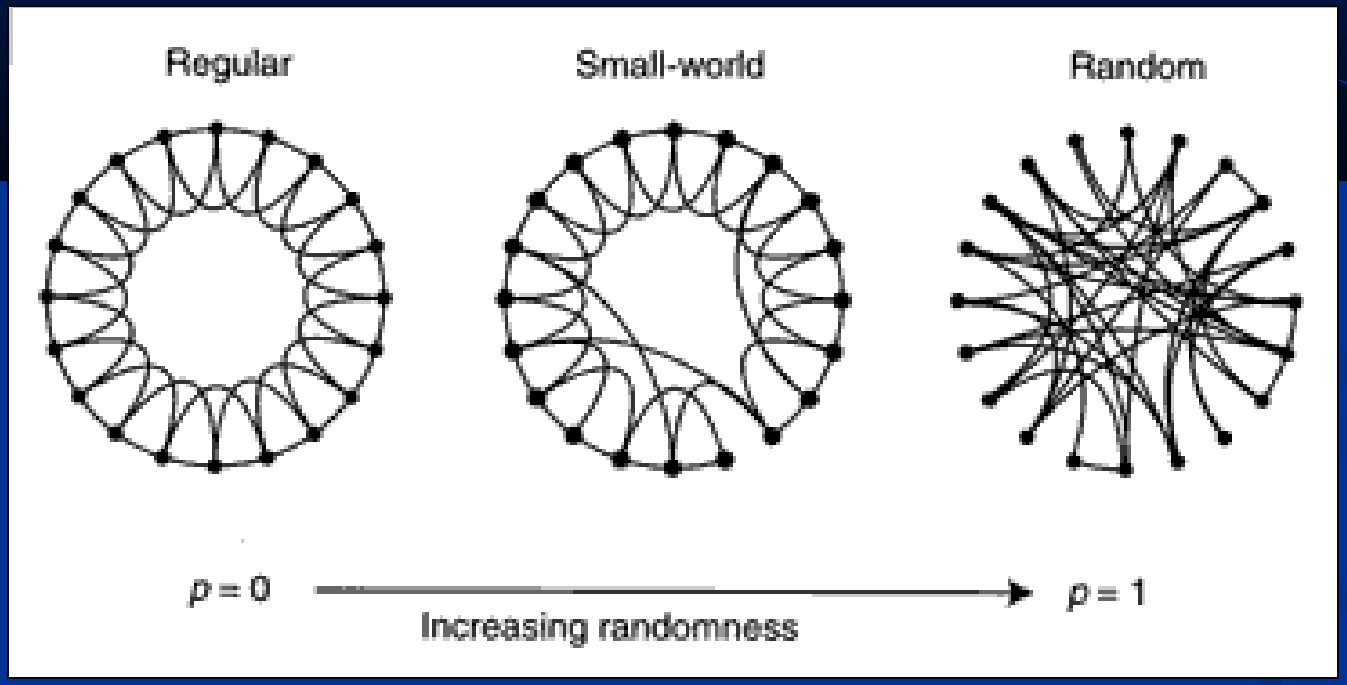
$$\langle \ell \rangle \sim \frac{\log(N)}{\log(z)}$$

Poisson distribution



Redes Small World

Duncan J. Watts & Steven H. Strogatz,
"Collective dynamics of 'small-world' networks". Nature 393: 440-442 (1998).



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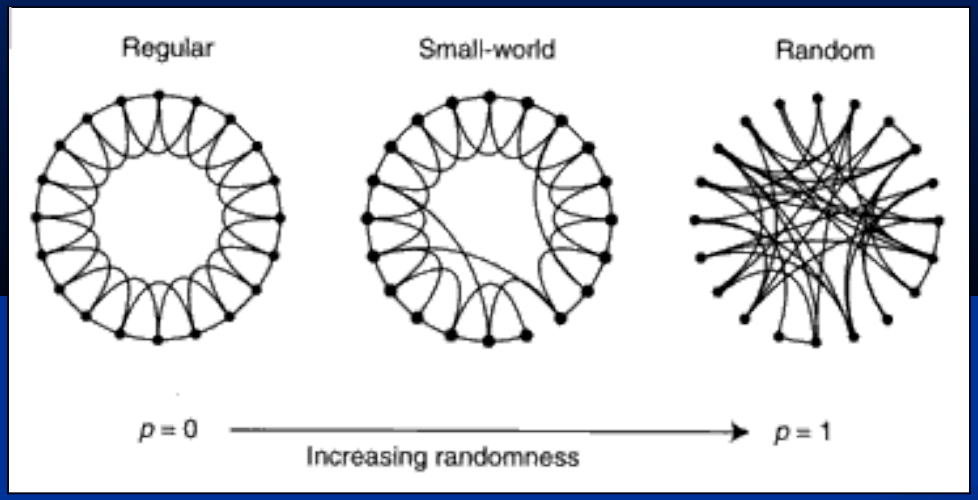

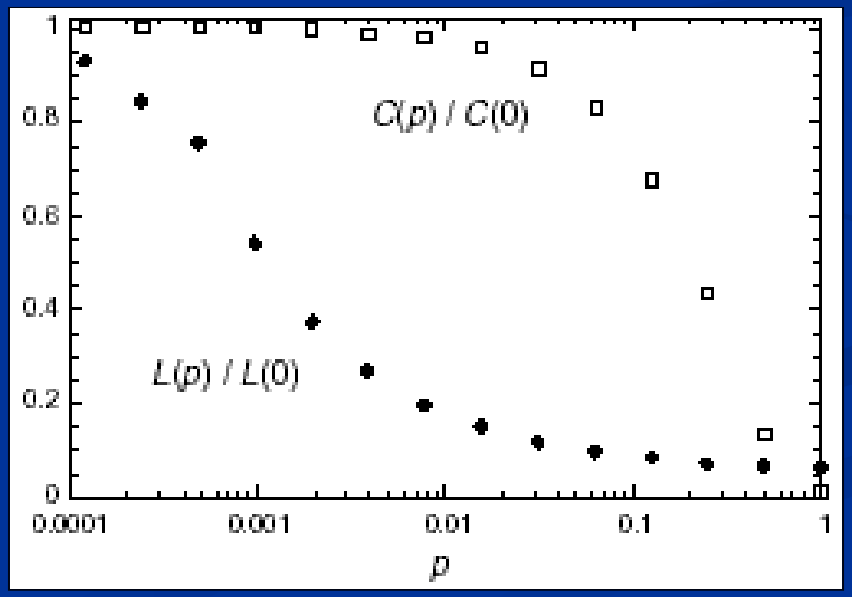

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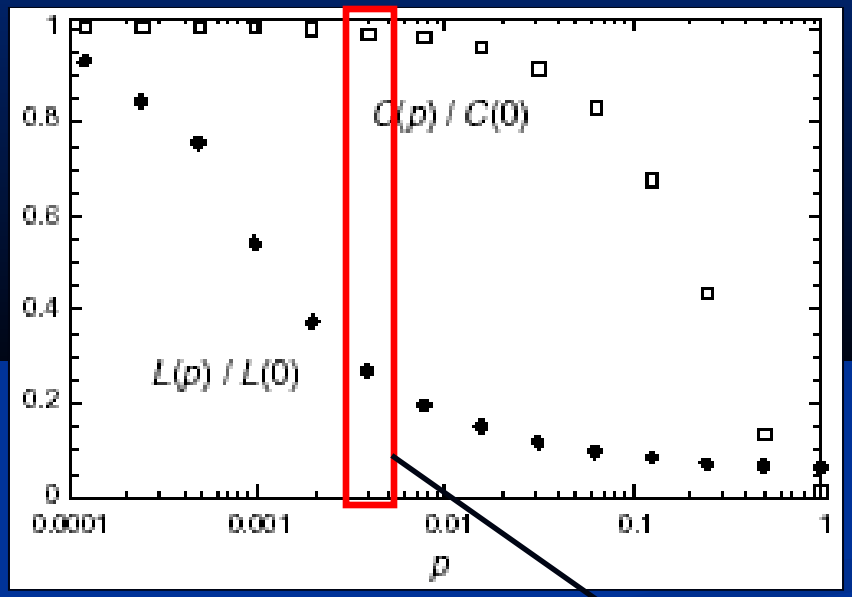
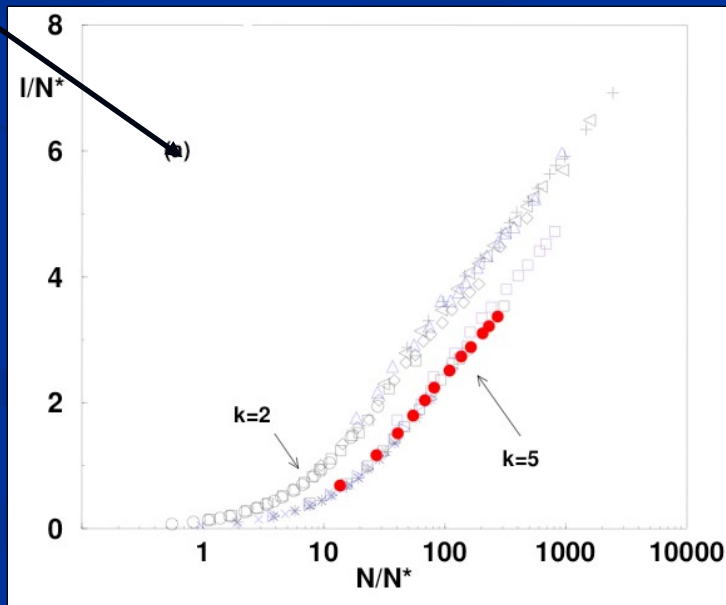
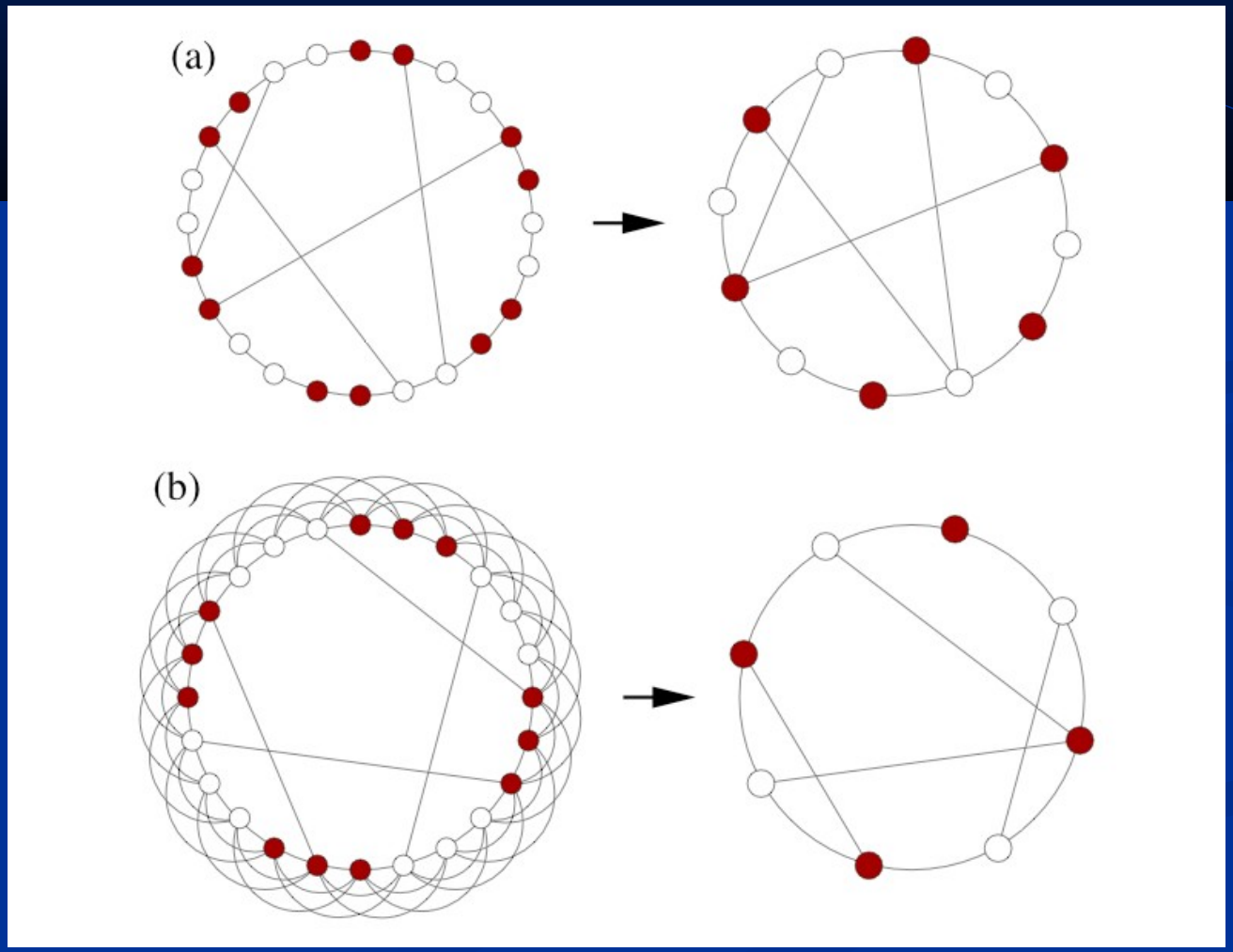




Grupo de renormalização e redes small-world:

``First-order phase transition on small-world networks",
M. Argollo de Menezes, C.F. Moukarzel and TJP Penna, Europhys. Lett. 50, 5 (2000)



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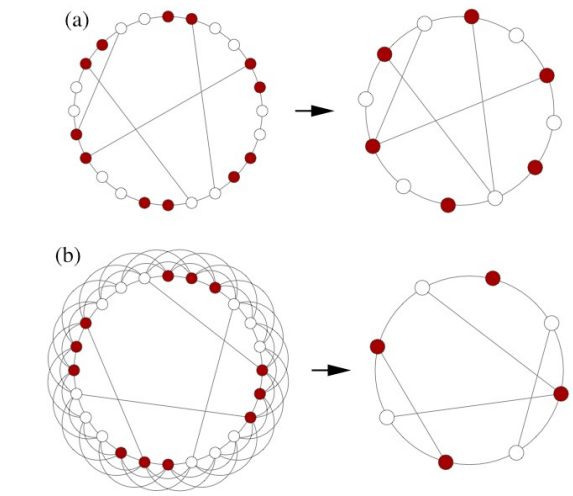
“No limite termodinâmico (N→∞) redes SW têm mesma propriedades topológicas de redes aleatórias (Erdös-Rényi)”



$$\log(1-p^{(N)}) = b^{(2d)} \log(1-p^{(bN)})$$

$$\ell(N,p) = L f\left(\frac{L}{L^*}\right)$$

$$f(x) \to A, \quad x \ll 1$$

$$f(x) \to \frac{\log(x)}{x}, \quad x \gg 1$$

p*=0 p*=1

$$L^* \sim p^{\tau}, \quad \tau = 1/d$$

A Internet

- 1957 - União Soviética lança com sucesso o Sputnik
- 1962 - DoD cria sistema de informação resistente a desastres (nucleares)

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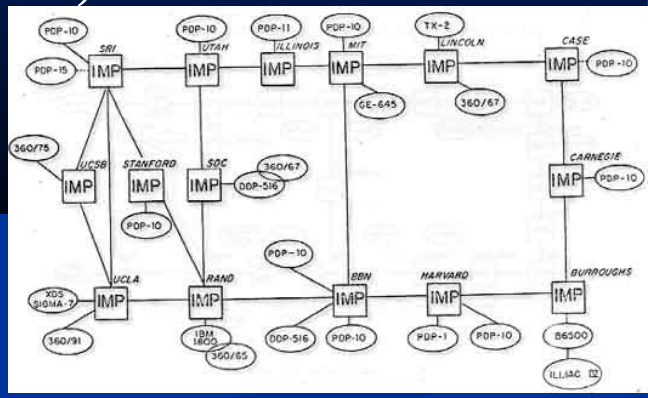
- Militares transferem sistema para universidades (ARPANET)



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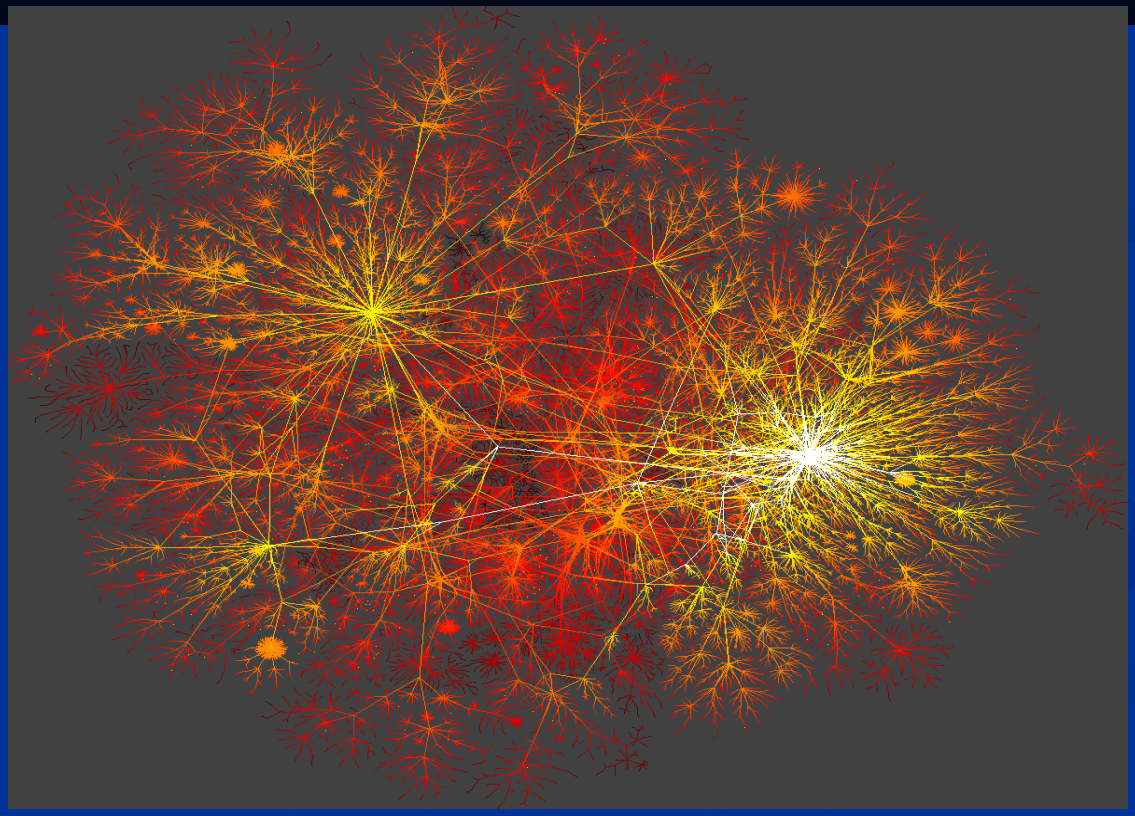
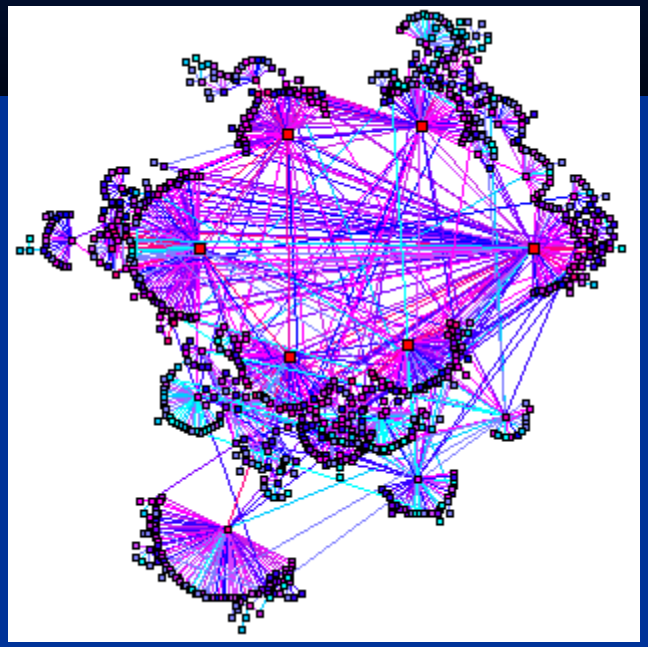
- NSFNET - Backbone + sub-redes (MCI, Sprint)
- Aumento exponencial no número de sub-redes



World Wide Web

Nodes: WWW documents
Links: URL links

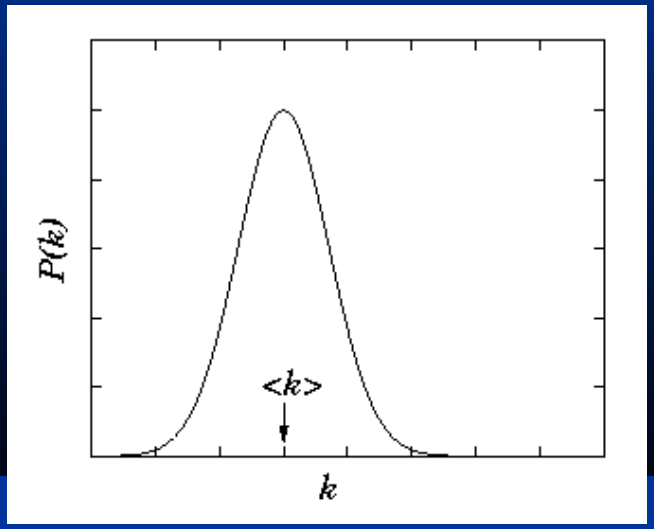
800 million documents
(S. Lawrence, 1999)



ROBOT: **collects all URL's found in a document and follows them recursively**

R. Albert, H. Jeong, A-L Barabasi, Nature, **401** 130 (1999)

What did we expect?



$\langle k \rangle \sim 6$

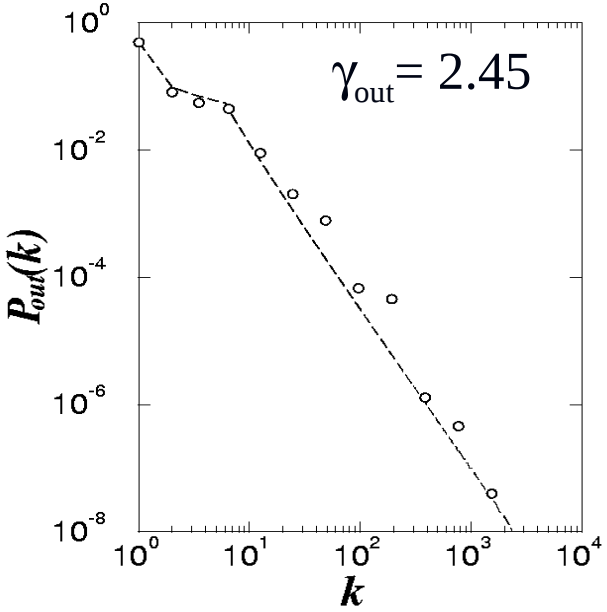
$P(k=500) \sim 10^{-99}$

$N_{www} \sim 10^{9}$

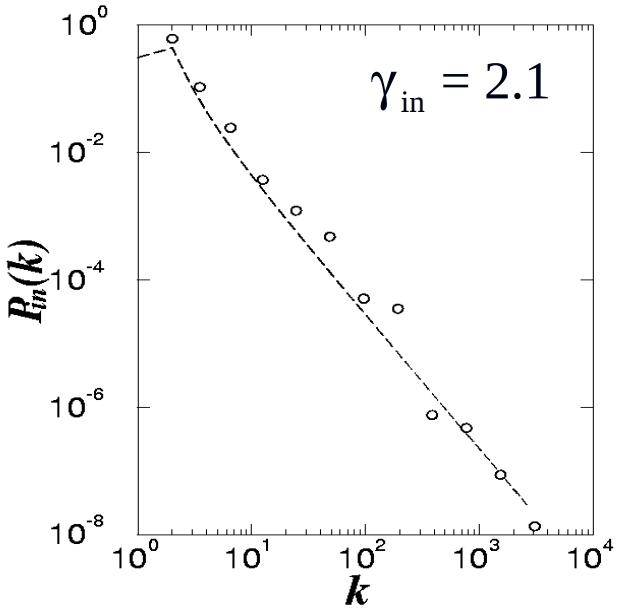
$\Rightarrow N(k=500) \sim 10^{-90}$

We find:

$$P_{out}(k) \sim k^{-\gamma_{out}}$$



$$P_{in}(k) \sim k^{-\gamma_{in}}$$



$P(k=500) \sim 10^{-6}$

$N_{www} \sim 10^{9}$

$\Rightarrow N(k=500) \sim 10^{3}$

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social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1	
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0	
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17
	student relationships	undirected	573	477	1.66	16.01	–	0.005
	sexual contacts	undirected	2 810				3.2	
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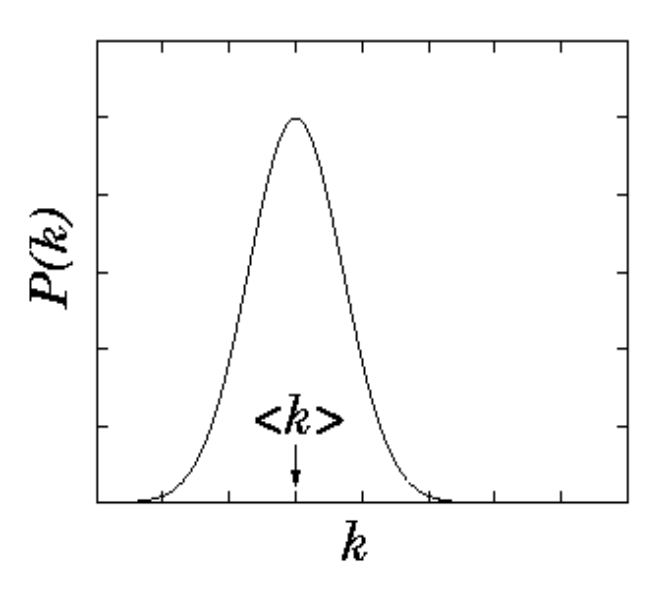
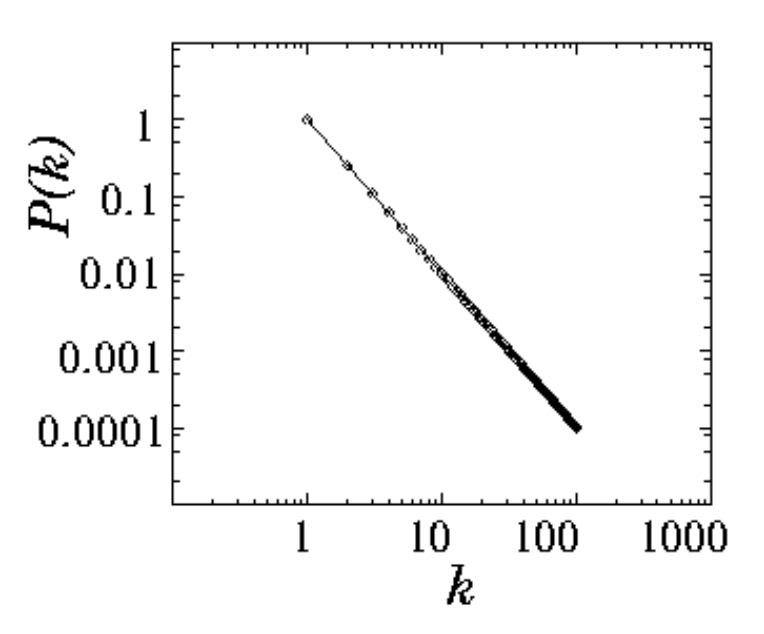
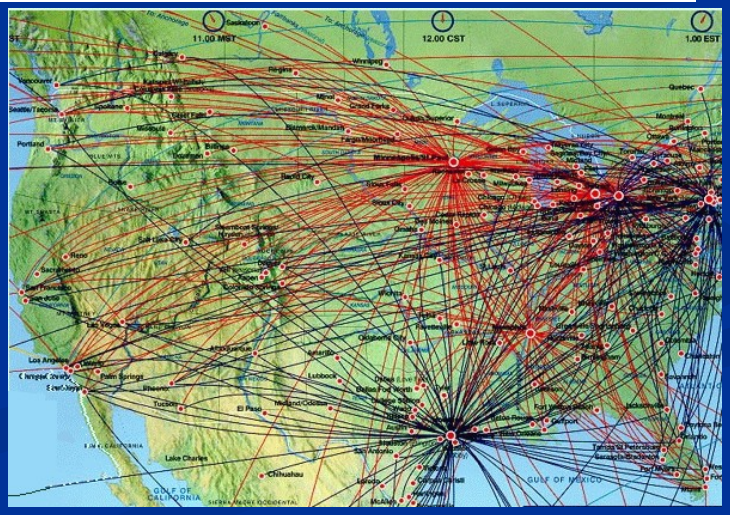
	network	type	n	m	z	ℓ	α	$C^{(1)}$
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1	
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0	
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17
	student relationships	undirected	573	477	1.66	16.01	–	0.005
	sexual contacts	undirected	2 810				3.2	
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7	
	citation network	directed	783 339	6 716 198	8.57		3.0/–	
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7	
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10
	train routes	undirected	587	19 603	66.79	2.16	–	
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072
	marine food web	directed	135	598	4.43	2.05	–	0.16
	freshwater food web	directed	92	997	10.84	1.90	–	0.20
	neural network	directed	307	2 359	7.68	3.97	–	0.18

O que isso significa?





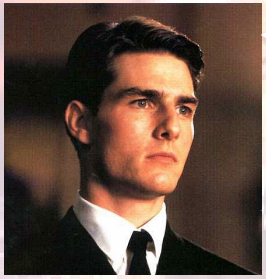


Exponential

Scale-free

ACTOR CONNECTIVITIES

Nodes: actors
Links: cast jointly



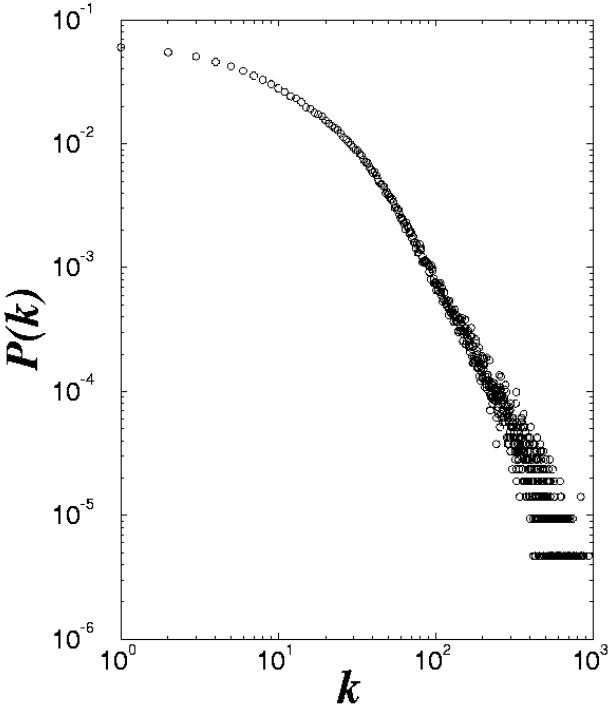
Days of Thunder (1990) Far and Away (1992) Eyes Wide Shut (1999)

N = 212,250 actors

⟨k⟩ = 28.78

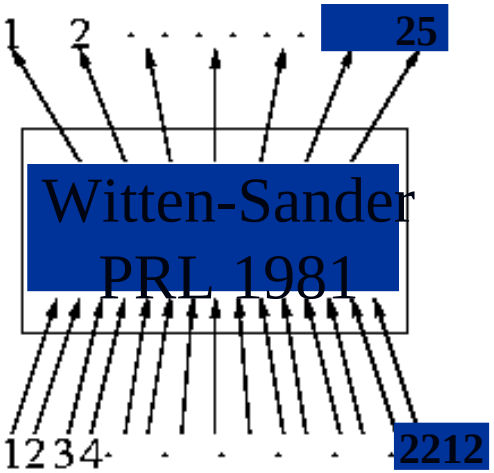
$P(k) \sim k^{-\gamma}$

$\gamma = 2.3$



SCIENCE CITATION INDEX

Nodes: papers
Links: citations



1736 PRL papers (1988)

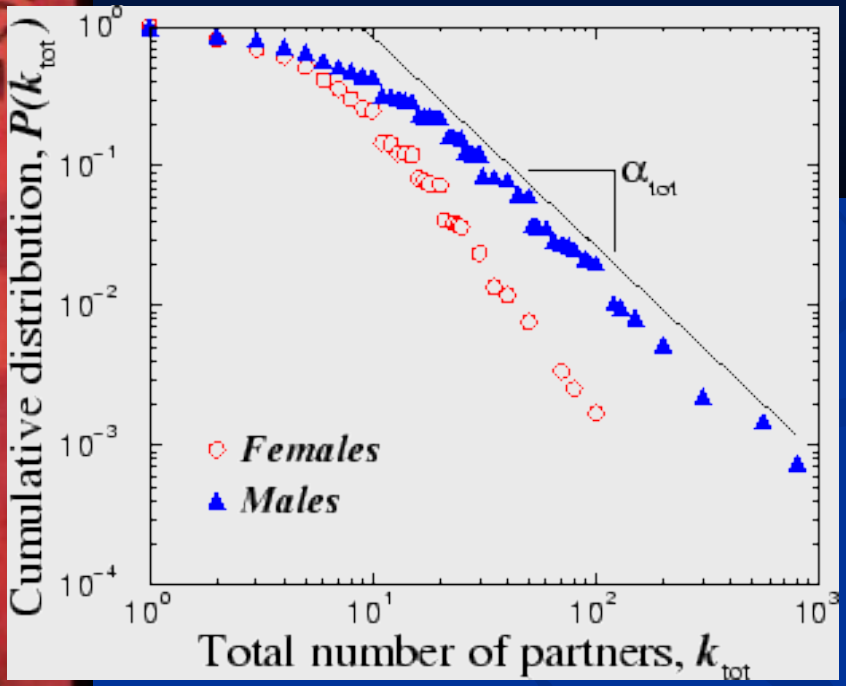
$$P(k) \sim k^{-\gamma}$$

$$(\gamma = 3)$$

(S. Redner, 1998)

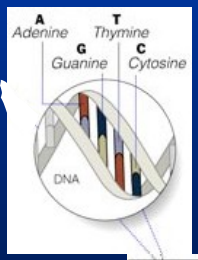
Sex-web

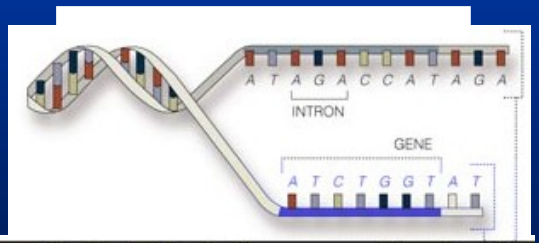
Nodes: people (Females; Males)
Links: sexual relationships



4781 Swedes; 18-74;
59% response rate.

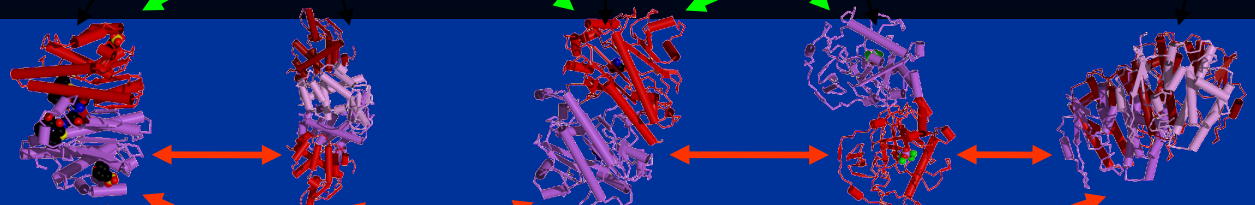
Liljeros et al. Nature 2001





GENOMA

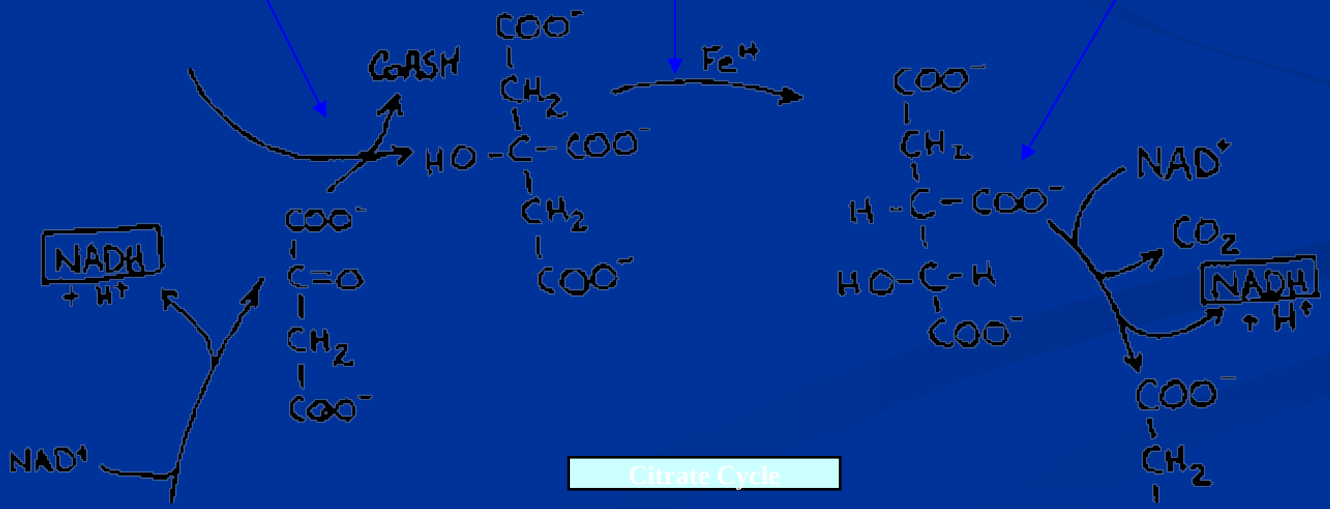
Interação proteina-gen



PROTEOMA

Interação proteina-proteina

enzimas



METABOLOMA

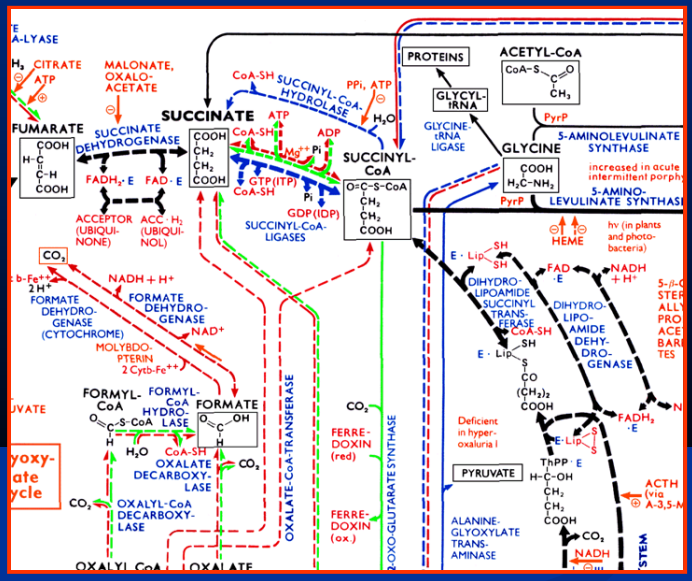
Reações bioquímicas

Citrate Cycle

Cadeias de reações metabólicas (em verdade, **redes**)

# Reação	Fórmula	Enzima
1	$2\ ATP + Ac \rightarrow AcCoA$	acs
2	$AcP \rightarrow ATP + Ac$	ackA
3	$AcP \rightarrow ATP + Ac$	ackE
4	$H_{ex} \rightarrow ATP$	atp_e
N	$41.257\ ATP + 3.547\ NAD + 0.205\ G6P \ldots \rightarrow Biomassa$	-

Cadeias de reações metabólicas (em verdade, **redes**)



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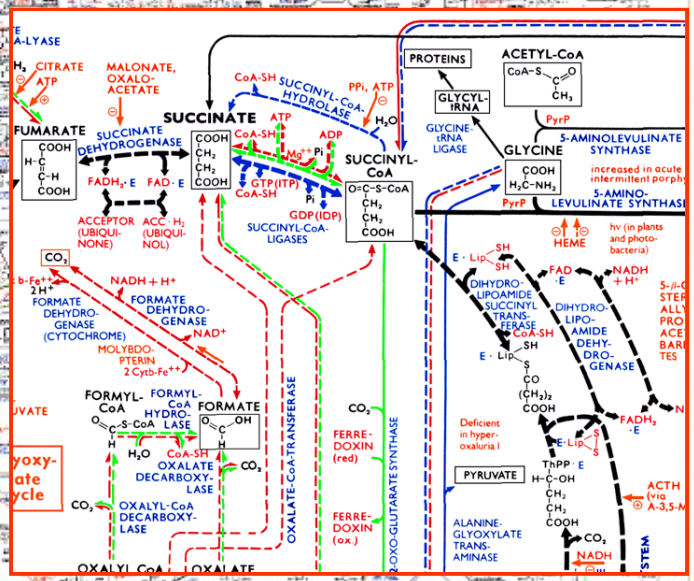
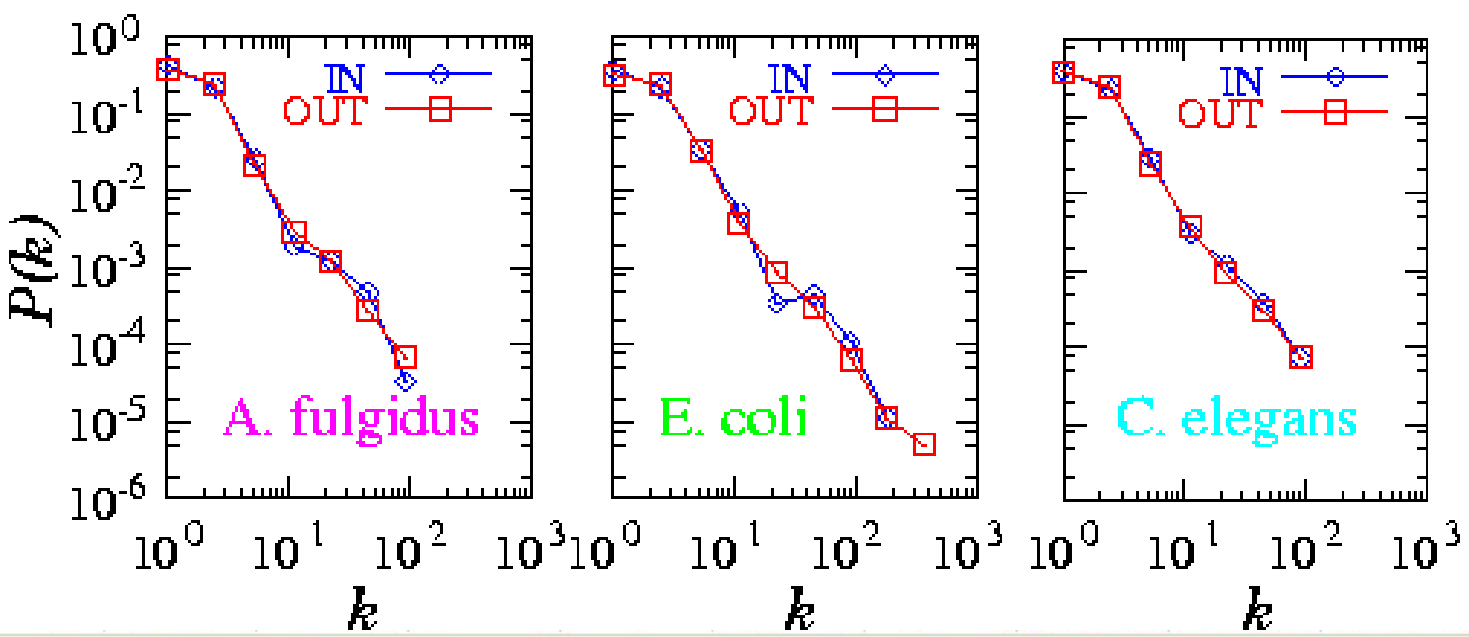
Biochemical Pathways



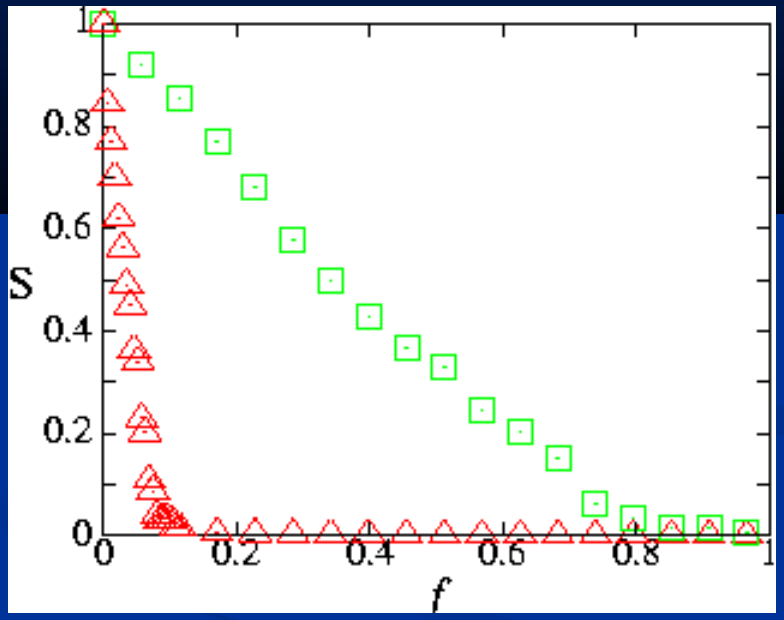
Biochemical Pathways



Vantagens e desvantagens da topologia scale-free

- Resistente ao malfuncionamento ocasional de seus elementos (sítios)
- Extremamente sensível a ataques intencionais dos Hubs.

Vantagens e desvantagens da topologia scale-free



- Resistente a uma falha ocasional de algum de seus elementos
- Extremamente sensível a ataques intencionais dos Hubs.

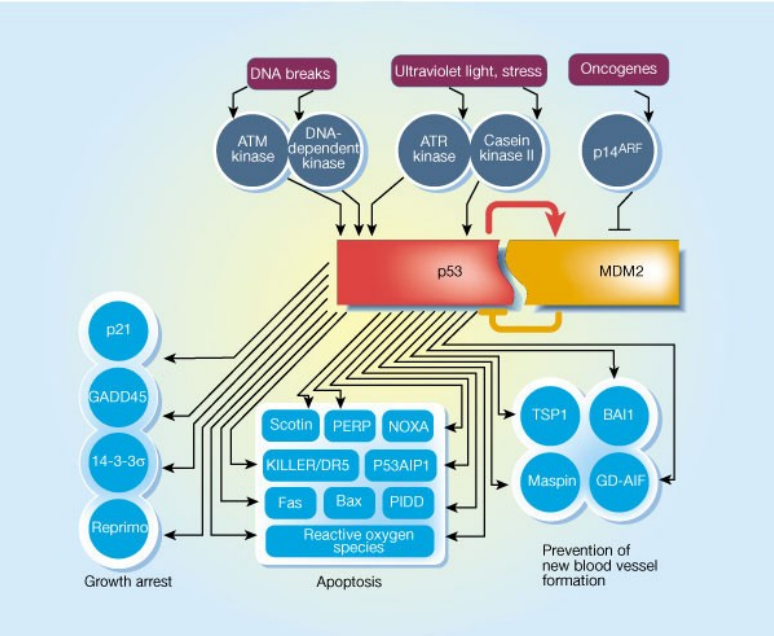
Nature **408** 307 (2000)

news and views feature

Surfing the p53 network

Bert Vogelstein, David Lane and Arnold J. Levine

The p53 tumour-suppressor gene integrates numerous signals that control cell life and death. As when a highly connected node in the Internet breaks down, the disruption of p53 has severe consequences.



…

"One way to understand the p53 network is to compare it to the Internet. The cell, like the Internet, appears to be a '**scale-free network**'."

Perguntas (V ou F, justifique)

1 ( ) Ao tratar sistemas com elementos interagentes como redes não perdemos informação

2 ( ) Os sítios de redes aleatórias possuem conectividades bastante distintas

3 ( ) Aeroportos são exemplos de topologias livre de escala

4 ( ) Redes livre de escala são resistentes a ataques intencionais a seus “hubs”